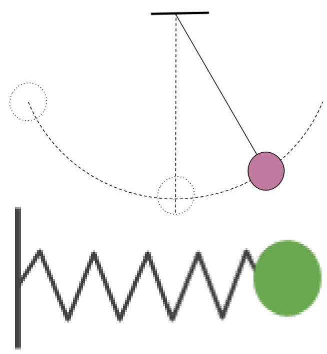
CAIE Physics A-level

Topic 17: Oscillations

Notes

17 - Oscillations (A-level only)

17.1 - Simple Harmonic Motion Oscillations



Free oscillations occur when no external force is continuously acting on the system, so its energy remains constant. The system will oscillate at its **natural frequency**.

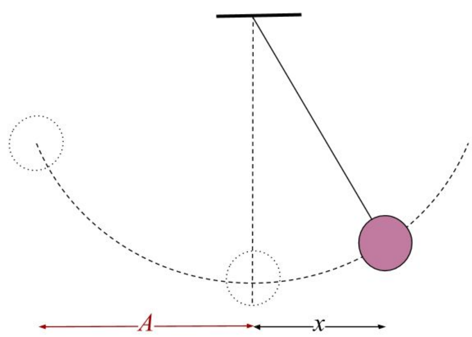
Examples of systems which experience **free oscillations** are:

- **Simple pendulum -** A small, dense bob that hangs from a string, which is attached to a fixed point. Once the bob is displaced and let go, the pendulum will oscillate freely.
- **Mass-spring system -** A mass attached to a spring, which will oscillate freely once displaced and released.
- **Tuning fork -** This will oscillate freely after being struck.

An object is experiencing **simple harmonic motion** when its acceleration is **directly proportional** to **displacement** and is in the **opposite direction**. These conditions can be shown through the equation:

$$a = -\omega^2 x$$

Where a is acceleration, ω is angular speed, x is displacement from the equilibrium position



An example of a simple harmonic oscillator is the **simple pendulum**, as shown in the diagram on the right.
The pendulum oscillates around a central midpoint known as the equilibrium position. Marked on the diagram by an x is the measure of **displacement**, and by an A is the **amplitude** of the oscillations - this is the maximum displacement. You could also measure the **time period (T)** of the oscillations by measuring the time taken by the pendulum to move from the equilibrium position, to the maximum displacement to the left, then to the maximum displacement to the right and back to the equilibrium position.

The **frequency (f)** is the **number of full oscillations** completed **per unit time**. You can calculate the frequency by finding the reciprocal of the time period (T) of an oscillation:

$$f = \frac{1}{T}$$

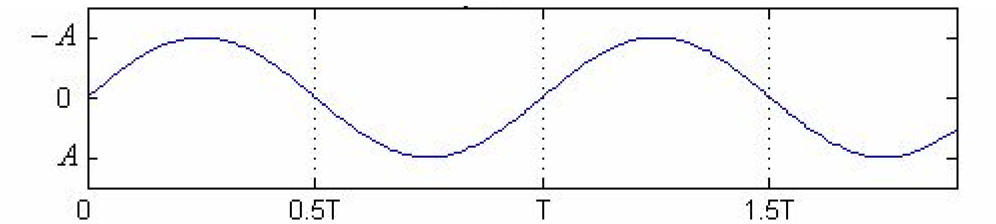
The **angular frequency (ω)** is the **angle** an object moves through **per unit time** (has only magnitude). You can calculate angular frequency by finding the product of frequency and 2π:

$$\omega = 2\pi f \qquad \omega = \frac{2\pi}{T} \quad \text{As } f = \frac{1}{T}$$

Phase difference is used to compare the stages that two oscillating objects are in. This is usually expressed as an angle (in degrees or radians) or as a fraction of a period. The degree by which objects are out of phase with each other is described by their phase difference:

- **In phase** - they're oscillating at the exact same time (meaning they must also have the same frequency) and their phase difference will be a multiple of 360° (2π radians).
- **Completely out of phase** - their oscillations are the exact opposites of each other and their phase difference will be an odd multiple of 180° (π radians).
- **90°/ 1/4 of a cycle out of phase** - this is where one object is oscillating a quarter of a cycle behind the other.

The motion of a simple harmonic oscillator can observed by using a **position sensor attached to a data logger**. Using the data logger you can form a graph of displacement against time as shown below:



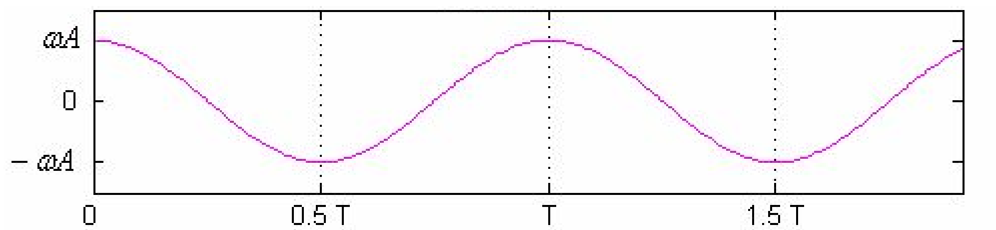
The exact displacement-time graph for a simple harmonic oscillator is described using the following equation:

$$x = x_0 \sin \omega t$$

Where x is the displacement, x_0 is the amplitude, ω is the angular frequency and t is the time.

The above equation is known as a **solution** to the defining equation of simple harmonic motion ($a = -\omega^2 x$).

As we know that **velocity is the derivative of displacement**, we can draw a **velocity-time graph** by drawing the **gradient function of the above graph**.



The exact velocity-time graph for a simple harmonic oscillator is described using the following equation (which is the derivative of the displacement equation):

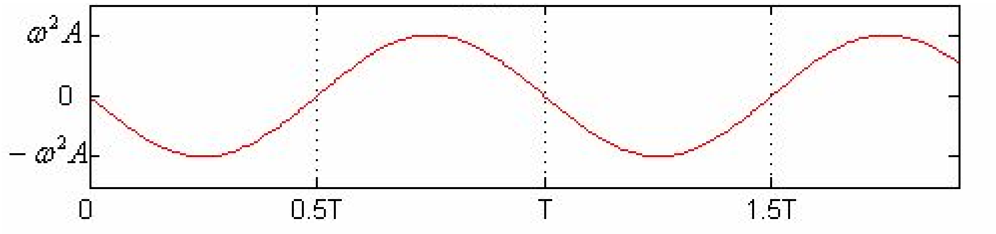
$$v = v_0 cos\omega t$$ where v_0 is equivalent to ωA/ωx_0

Another equation which describes the velocity of a simple harmonic oscillator is:

$$v = \pm \omega \sqrt{(x_0{}^2 - x^2)}$$

Note that the maximum and minimum velocity on the graph occurs when displacement is 0, as predicted in the above equation.

Finally, we know **acceleration is the derivative of velocity**, so we can draw an **acceleration-time graph** by drawing the **gradient function of the above graph**.



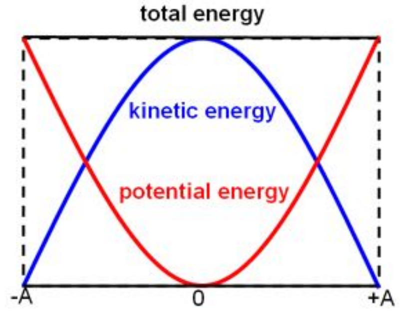
By comparing the displacement-time and acceleration-time graphs, you can clearly see the defining characteristic of simple harmonic motion: acceleration is directly proportional and in the opposite direction to displacement. In simple harmonic motion, acceleration and displacement are **completely (180°) out of phase with each other**.

Looking at the graphs of displacement and velocity against time - you can see that they are **90° out of phase**.

17.2 - Energy in Simple Harmonic Motion

For any simple harmonic motion system, **kinetic energy is transferred to potential energy and back as the system oscillates**. The type of potential energy depends on the system.

At the **amplitude** of its oscillations, the system will have the maximum amount of potential energy. As it moves towards the equilibrium position, this potential energy is converted to kinetic energy so that at the **centre of its oscillations** the kinetic energy is at a maximum. As the system then moves away from the equilibrium again, the kinetic energy is transferred to potential energy until it is at a maximum again and this process repeats for one full oscillation. **The total energy of the system remains constant** (when air resistance is negligible, otherwise energy is lost as heat).



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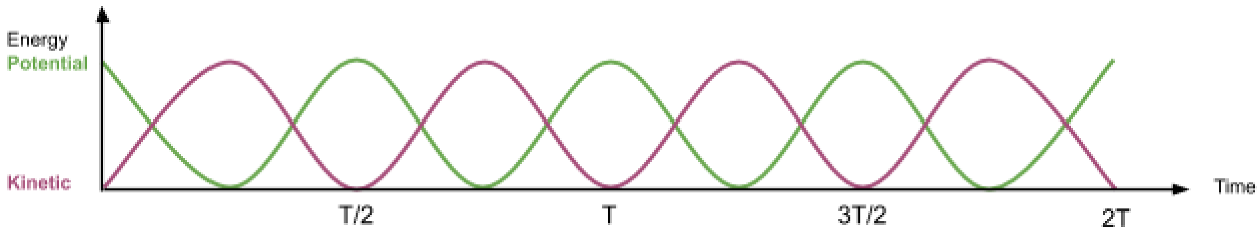
The total energy of a system undergoing simple harmonic motion is given by

$$E = \frac{1}{2} m\omega^2 {x_0}^2$$

This figure will be a constant and the sum of the kinetic and potential energies at any given point in the oscillation,

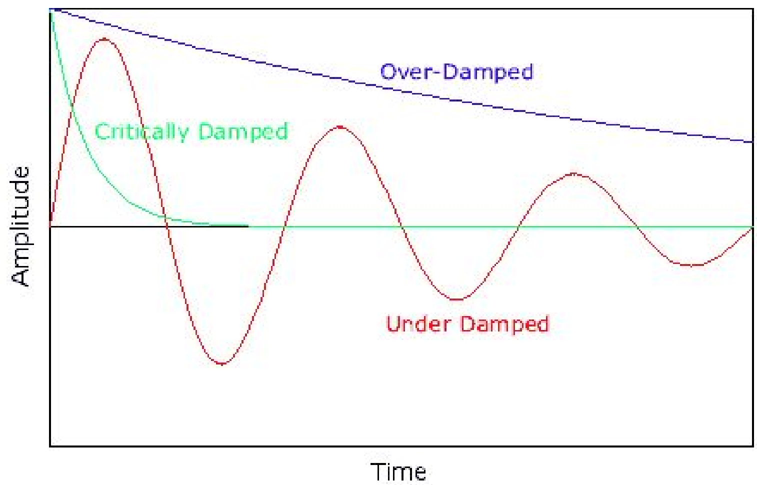
The diagram below shows the **variation of energy with displacement**, while the diagram below shows the **variation of energy with time**, for a simple harmonic system staring at its amplitude.



17.3 - Damped and forced oscillations, resonance

Damping is where the energy in an oscillating is lost to the environment, leading to reduced amplitude of oscillations. There are 3 main types of damping:

- **Light damping** - This is also known as under-damping and this is where the amplitude gradually decreases by a small amount each oscillation.
- **Critical damping** - This reduces the amplitude to zero in the shortest possible time (without oscillating).
- **Heavy damping** - This is also known as over-damping, and is where the amplitude reduces slower than with critical damping, but also without any additional oscillations.

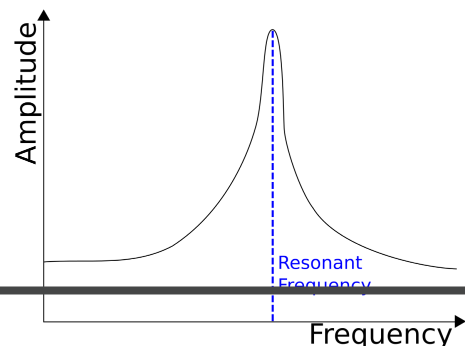


An example of a system experiencing **light damping** is a **simple pendulum** experiencing damping due to **air resistance**: after each oscillation the maximum amplitude of the pendulum decreases slightly. The amplitude of the oscillations follows an **exponential decay**.

Critical damping is incredibly important when the amplitude must be reduced to zero in the fastest possible time, without oscillating. **Shock absorbers** are devices which allow the oscillations experienced by a car suspension to be critically damped, making controlling the car much easier. **Measurement instruments** such as speedometers are also critically damped so that their pointers do not oscillate, and move to the correct position quickly to avoid confusion.

An example of a **heavy damping** device is a **door closer**, which allows a door to slowly close without oscillating.

Forced vibrations are where a system experiences an **external driving force** which causes it to oscillate, the frequency of this driving force, known as driving frequency, is significant. If the



driving frequency is equal to the natural frequency of a system (also known as the resonant frequency), then **resonance** occurs.

As the driving frequency approaches the resonant frequency, the amplitude of oscillations will increase, as shown in the graph to the right.

Resonance is where the amplitude of oscillations of a system drastically increase due to gaining an increased amount of energy from the driving force.

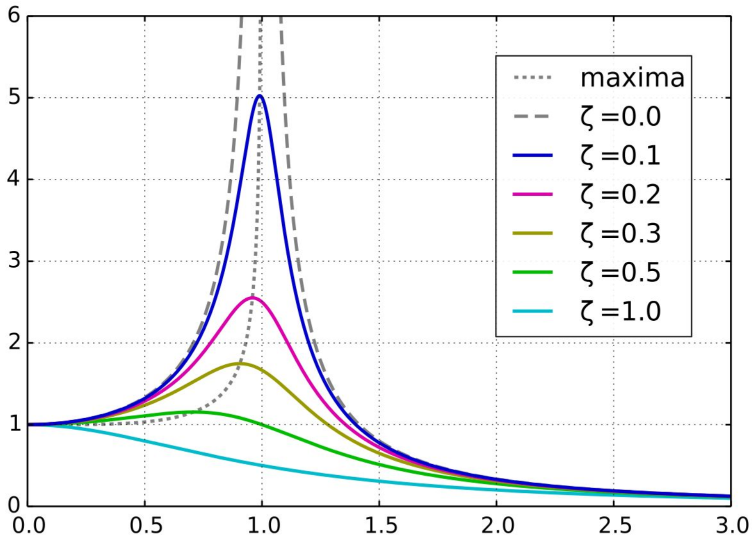
Resonance has many useful applications for example:

- **Instruments** - An instrument such as a flute has a long tube in which air resonates, causing a stationary sound wave to be formed.
- **Radio** - These are tuned so that their electric circuit resonates at the same frequency as the desired broadcast frequency.
- **Swing** - If someone pushes you on a swing they are providing a driving frequency, which can cause resonance if it's equal to the resonant frequency, causing you to swing higher.

However, resonance also causes many undesirable effects in:

- **Bridges** - People travelling across a bridge will provide a driving force which, when equal to the natural frequency, will cause large oscillations due to resonance. These oscillations are potentially dangerous and could lead to the damage of the bridge.
- **Aircraft** - Parts of the aircraft may experience resonance, which causes large oscillations that can lead to those parts being damaged.

Damping can be used to decrease the effect of resonance. Different types of damping will have different effects: **as the degree of damping increases**, the resonant frequency decreases (shifts to left on a graph), the maximum amplitude decreases and the peak of maximum amplitude becomes wider. These effects are shown in the graph below, where ζ is the damping ratio, $\zeta = 1$ represents critical damping.



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You can see from the diagram above that the **sharpness** of the resonance **decreases** as the degree of damping **increases**.

The sharpness of the resonance is a measure of how quickly the amplitude of the oscillations decays as you move either side of the peak.